Statistics Probability Examples And Solutions

INTRODUCTION TO STATISTICS PROBABILITY EXAMPLES AND SOLUTIONS

Probability is the mathematical backbone of uncertainty. Whether you are forecasting weather, analyzing financial markets, or interpreting medical test results, probability helps you make sense of randomness and data. For students, researchers, and professionals alike, mastering statistics probability examples and solutions is essential for building a strong foundation in data analysis. This article provides a clear, step-by-step exploration of core probability concepts, complete with practical examples and fully worked solutions. By the end, you will not only understand the theory but also know how to apply it to real-world problems.

WHAT IS

PROBABILITY IN STATISTICS?

In the broadest sense, probability is a numerical measure of how likely an event is to occur. It ranges from 0 (impossible) to 1 (certain). In statistics, probability is used to quantify uncertainty and to make inferences about populations based on samples. The fundamental formula for probability is:

P(Event) = Number of Favorable Outcomes / Total Number of Possible Outcomes

This simple formula forms the basis for countless statistics probability examples and solutions across disciplines. Before diving into complex problems, it is crucial to understand key terminology: experiment, outcome, sample space, and event. An experiment is any process that generates outcomes. The sample space is the set of all possible outcomes. An event is any subset of the sample space that we are interested in.

TYPES OF PROBABILITY

There are three main approaches to defining probability, each useful in different scenarios:

- **Theoretical (Classical) Probability** – Based on the assumption that all outcomes are equally likely. For example, the probability of rolling a 4 on a fair six-sided die is 1/6.
- **Empirical**

(Experimental) Probability – Based on observed data from experiments or historical records. For instance, if you flip a coin 100 times and get 53 heads, the empirical probability of heads is 0.53.
- **Subjective Probability** – Based on personal judgment or expert opinion. This is often used when data is scarce, such as in forecasting the success of a new product launch.

Understanding these types is essential when working through statistics probability examples and solutions, because the method you use to calculate probability depends on the context and available data.

BASIC PROBABILITY RULES AND FORMULAS

To solve probability problems efficiently, you must be comfortable with a few fundamental rules. These rules appear repeatedly in statistics probability examples and solutions at every level.

The Addition Rule

The addition rule is used to find the probability that either event A or event B (or both) occurs.

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

If events are mutually exclusive (they cannot happen at the same time), the formula simplifies to: **P(A ∪ B) = P(A) + P(B)**.

The Multiplication Rule

The multiplication rule is used to find the probability that both event A and event B occur.

P(A ∩ B) = P(A) × P(B | A)

If events are independent (the occurrence of one does not affect the other), the formula simplifies to: **P(A ∩ B) = P(A) × P(B)**.

Conditional Probability

Conditional probability is the probability of event A occurring given that event B has already occurred.

P(A | B) = P(A ∩ B) / P(B)

This is one of the most powerful concepts in probability, appearing in fields from medicine to machine learning.

Complement Rule

The complement of an event A is the event that A does not occur.

P(A') = 1 - P(A)

STATISTICS PROBABILITY EXAMPLES AND SOLUTIONS: BEGINNER LEVEL

Let us begin with some straightforward examples that illustrate the basic principles. These simple statistics probability examples and solutions will build your confidence before we move to more advanced topics.

Example 1: Rolling a Die

Problem: A fair six-sided die is rolled. What is the probability of rolling an even number?

Solution: The sample space is {1, 2, 3, 4, 5, 6}. The favorable outcomes (even numbers) are {2, 4, 6}. There are 3 favorable outcomes and 6 total outcomes. So, P(even) = 3/6 = 1/2 = 0.5. There is a 50% chance of rolling an even number.

Example 2: Drawing a Card from a Standard Deck

Problem: A card is drawn at random from a standard 52-card deck. What is the probability of drawing a heart?

Solution: There are 13 hearts in a deck of 52 cards. P(heart) = 13/52 = 1/4 = 0.25. The probability is 25%.

Example 3: Tossing Two Coins

Problem: Two fair coins are tossed. What is the probability of getting exactly one head?

Solution: The sample space is {HH, HT, TH, TT}. The favorable outcomes (exactly one head) are {HT, TH}. So, P(exactly one head) = 2/4 = 1/2 = 0.5.

These foundational statistics probability examples and solutions demonstrate how to identify the sample space and count favorable outcomes. As problems grow more complex, systematic counting methods and probability rules become indispensable.

STATISTICS PROBABILITY EXAMPLES AND SOLUTIONS: INTERMEDIATE LEVEL

Now we will tackle problems that require the use of the addition and multiplication rules. These intermediate statistics probability examples and solutions will help you handle overlapping events and dependent scenarios.

Example 4: Drawing Cards with Replacement (Independent Events)

Problem: Two cards are drawn from a standard 52-card deck with replacement. What is the probability that both cards are kings?

Solution: Since the card is replaced, the draws are independent. P(king on first draw) = 4/52 = 1/13. P(king on second draw) = 4/52 = 1/13. Using the multiplication rule for independent events: P(both kings) = (1/13) × (1/13) = 1/169 ≈ 0.0059, or about 0.59%.

Example 5: Drawing Cards Without Replacement

(Dependent Events)

Problem: Two cards are drawn from a standard 52-card deck without replacement. What is the probability that both cards are aces?

Solution: P(ace on first draw) = 4/52 = 1/13. After drawing one ace, there are 3 aces left and 51 cards remaining. P(ace on second draw | ace on first) = 3/51 = 1/17. Using the multiplication rule: P(both aces) = (4/52) × (3/51) = 12/2652 = 1/221 ≈ 0.0045, or about 0.45%.

Example 6: Using the Addition Rule

Problem: A single card is drawn from a standard deck. What is the probability that it is a heart or a king?

Solution: Let A be the event "card is a heart" and B be the event "card is a king." P(A) = 13/52 = 1/4. P(B) = 4/52 = 1/13. The intersection (heart and king) is the king of hearts, so P(A ∩ B) = 1/52. Using the addition rule: P(A ∪ B) = (13/52) + (4/52) – (1/52) = 16/52 = 4/13 ≈ 0.3077, or 30.77%.

These statistics probability examples and solutions illustrate how events can be independent or dependent, and how the addition rule prevents double-counting when events overlap.

STATISTICS PROBABILITY EXAMPLES AND SOLUTIONS: ADVANCED LEVEL

Advanced problems often involve conditional probability, Bayes' theorem, and probability distributions. These statistics probability examples and solutions are common in upper-level statistics courses and professional data analysis.

Example 7: Conditional Probability in Medical Testing

Problem: A certain disease affects 1% of the population. A test for the disease has a 99% sensitivity (true positive rate) and a 95% specificity (true negative rate). If a randomly selected person tests positive, what is the probability that they actually have the disease?

Solution: This is a classic application of Bayes' theorem. Let D be the event "has the disease" and T be the event "tests positive." We want P(D | T).

- P(D) = 0.01 (prevalence)
- P(T | D) = 0.99 (sensitivity)
- P(not D) = 0.99
- P(T | not D) = 1 – 0.95 = 0.05 (false positive rate)

Using Bayes' theorem: P(D | T) = [P(T | D) × P(D)] / [P(T | D) × P(D) + P(T | not D) × P(not D)]

P(D | T) = (0.99 × 0.01) / (0.99 × 0.01 + 0.05 × 0.99) = 0.0099 / (0.0099 + 0.0495) = 0.0099 / 0.0594 ≈ 0.1667, or about 16.67%.

This result surprises many people: even with a highly accurate test, a positive result does not guarantee the disease is present, especially when the disease is rare. This example shows why understanding conditional probability is critical in healthcare and diagnostics.

Example 8: Probability Distribution – Binomial

Distribution

Problem: A fair coin is tossed 10 times. What is the probability of getting exactly 7 heads?

Solution: This follows a binomial distribution with n = 10 trials, success probability p = 0.5, and we want k = 7 successes. The binomial probability formula is: P(X = k) = C(n, k) × p^k × (1-p)^(n-k), where C(n, k) = n! / [k! (n-k)!].

C(10, 7) = 120. p^7 = 0.5^7 = 1/128. (1-p)^3 = 0.5^3 = 1/8. So P(X=7) = 120 × (1/128) × (1/8) = 120 / 1024 = 15/128 ≈ 0.1172, or about 11.72%.

Example 9: Probability Distribution – Normal Distribution

Problem: The heights of adult women in a certain population are normally distributed with a mean of 165 cm and a standard deviation of 6 cm. What is the probability that a randomly selected woman is taller than 177 cm?

Solution: First, compute the z-score: $z = (X - \mu) / \sigma = (177 - 165) / 6 = 12/6 = 2.00$. Using the standard normal distribution table, the probability of being less than z = 2.00 is about 0.9772. Therefore, the probability of being taller than 177 cm is 1 – 0.9772 = 0.0228, or about 2.28%.

These advanced statistics probability examples and solutions demonstrate the power of distribution models. The binomial distribution is ideal for counting successes in independent trials, while the normal distribution is the cornerstone of inferential statistics.